

Behavior of universal critical parameters in the QCD phase diagram

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Abstract. We determine the dependence of important parameters for critical fluctuations on temperature and baryon chemical potential in the QCD phase diagram. The analysis is based on an identification of the fluctuations of the order parameter obtained from the Ising model equation of state and the Ginzburg-Landau effective potential approach. The impact of the mapping from Ising model variables to QCD thermodynamics is discussed.

1. Introduction

Effective models for QCD thermodynamics [1, 2, 3] predict that the chiral phase transition in QCD is of first order at large baryon chemical potential μ_B . At $\mu_B = 0$, lattice QCD simulations established that the transformation between the denser deconfined quark matter phase at high temperature T and the low- T hadronic matter phase is an analytic crossover [4]. As a consequence, the first-order phase transition line should terminate in a second-order chiral critical point at some non-zero μ_B . A critical point is characterized through the growth of the fluctuations of the order parameter σ in the scaling region around it. In the thermodynamic limit, these fluctuations diverge at the critical point with a given exponent of the diverging correlation length ξ .

Experimentally, the phases of QCD matter are studied in heavy-ion collision experiments by varying the beam energy and the system size. The quest for finding the conjectured critical point in the QCD phase diagram has triggered an extensive effort culminating in the recently completed beam energy scan phase I at RHIC [5, 6, 7] and being continued with the NA61 experiment at CERN-SPS and the phase II of RHIC's beam energy scan program in the future. Signaling the presence of a critical point, large event-by-event fluctuations [8, 9] of conserved quantities such as electric charge or baryon number have been predicted. In reality, the actual growth of the correlation length is limited by the finite size and, more importantly, lifetime of the created system. Thus, higher-order non-Gaussian cumulants of the event-by-event distributions, which depend more strongly on the growth of ξ [10], are particularly interesting. These exhibit certain patterns in the QCD phase diagram [11, 12] which are governed by the qualitative behavior of universal critical parameters.

Assuming that QCD belongs to the same universality class as the three-dimensional Ising model [2, 3] allows one to identify the expectation value of the order parameter of the chiral phase transition with the order parameter in the spin model, the magnetization M . Fluctuations of the order parameter in the scaling region can then be determined from the critical equation of state for M . This enables us to study the behavior of the universal critical parameters in this work. A similar approach was previously used in [13, 14] to construct an equation of state with critical point for QCD matter.

In the following, we employ the linear parametric representation [15] of the critical equation of state for the magnetization with critical exponents $\beta = 1/3$ and $\delta = 5$

$$M = M_0 R^\beta \theta, \quad (1)$$

$$r = R(1 - \theta^2), \quad (2)$$

$$h = R^{\beta\delta} \tilde{h}(\theta). \quad (3)$$

In this representation $M(r, h)$, which is a function of reduced temperature $r = (T - T_c)/T_c$ with spin model critical temperature T_c and of reduced external magnetic field $h = H/H_0$, is expressed in terms of the auxiliary variables $R \geq 0$ and θ as $M(R, \theta)$. The parametrization is uniquely defined within $-\theta_0 \leq \theta \leq \theta_0$, where for $\tilde{h}(\theta) = 3\theta(1 - 2\theta^2/3)$, which is an odd function of θ , one finds $\theta_0 = \sqrt{3/2}$ as the relevant non-trivial root. M_0 and H_0 are normalization constants of mass dimension one and three, respectively.

In spin model coordinates, the critical point is located at $r = h = 0$ ($R = 0$). At $h = 0$, one finds a first-order phase transition for $r < 0$ ($R > 0$ and $\theta = \pm\theta_0$), while for $r > 0$ there is a crossover ($R > 0$ and $\theta = 0$). The behavior of the order parameter in the scaling region is such that $M(r = 0, h) \sim |h|^{1/\delta} \text{sgn}(h)$ and $M(r, h \rightarrow 0^+) \sim |r|^\beta$ for $r < 0$, which can be assured by imposing conditions on M_0 and H_0 such that M is positive for $h \geq 0^+$ [13].

2. Order parameter fluctuations

The equilibrium cumulants quantifying the fluctuations of the order parameter in the scaling region are determined from derivatives

$$\langle (\delta\sigma)^n \rangle_c = \left(\frac{T}{VH_0} \right)^{n-1} \left. \frac{\partial^{n-1} M}{\partial h^{n-1}} \right|_r \quad (4)$$

of M with respect to the external magnetic field at fixed r . Explicit expressions in terms of R and θ based on the linear parametric representation can be found in [16]. For given r and h , these cumulants can then be determined by making use of Eqs. (2) and (3).

The cumulants of the fluctuations of the order parameter can, likewise, be determined from the corresponding probability distribution which depends on an effective action for the order parameter of Ginzburg-Landau type [16, 10]. Including up to quartic interaction terms one finds

$$\langle (\delta\sigma)^2 \rangle = \frac{T}{V} \xi^2, \quad (5)$$

$$\langle (\delta\sigma)^3 \rangle = -2\lambda_3 \frac{T^2}{V^2} \xi^6, \quad (6)$$

$$\langle (\delta\sigma)^4 \rangle_c = 6 \left(2(\lambda_3 \xi)^2 - \lambda_4 \right) \frac{T^3}{V^3} \xi^8. \quad (7)$$

In the scaling region, the cubic and quartic interaction strengths depend on the correlation length as $\lambda_3 = \tilde{\lambda}_3 T(T\xi)^{-3/2}$ and $\lambda_4 = \tilde{\lambda}_4 (T\xi)^{-1}$, where we neglected small anomalous scaling dimension corrections in the cumulant expressions.

Strictly speaking, the expressions in Eqs. (5) - (7) are only valid in the scaling region as long as the correlation length is small compared to the macroscopic length scale of the considered system. Keeping this limitation in mind we can, nonetheless, identify the expressions for the cumulants following from Eq. (4) with those in Eqs. (5) - (7). This yields parametric expressions for ξ , $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ in terms of the auxiliary spin model variables reading

$$\xi = \sqrt{\frac{M_0}{H_0}} \frac{1}{R^{2/3}(3 + 2\theta^2)^{1/2}}, \quad (8)$$

$$\tilde{\lambda}_3 = \frac{2\theta(9 + \theta^2)}{(3 - \theta^2)(3 + 2\theta^2)^{3/4}} C, \quad (9)$$

$$\tilde{\lambda}_4 = 2\tilde{\lambda}_3^2 + 2 \frac{(81 - 783\theta^2 + 105\theta^4 - 5\theta^6 + 2\theta^8)}{(3 - \theta^2)^3(3 + 2\theta^2)^{3/2}} C^2, \quad (10)$$

where $C = (T^2 H_0 / M_0^5)^{1/4}$. The parameters $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ are found to be dimensionless and independent of R which implies that they remain finite in the entire domain of the parametric representation, while ξ is of inverse mass dimension and diverges as $R \rightarrow 0$.

According to Eq. (9), $\tilde{\lambda}_3$ is an odd function of h which is negative for $h < 0$. At $h = 0$, one finds $\tilde{\lambda}_3 = 0$ for $r > 0$, while for $r < 0$ the dimensionless parameter approaches $\tilde{\lambda}_3 \rightarrow \pm 7C/6^{1/4}$ as $h \rightarrow 0^\pm$. The qualitative behavior of the fourth-order cumulant $\langle(\delta\sigma)^4\rangle_c$ is determined by the behavior of the parameter difference $2\tilde{\lambda}_3^2 - \tilde{\lambda}_4$. As evident from Eq. (10), this difference is an even function of h , which is positive for all $r < 0$ with maximum value $(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4) = 128C^2/27$ at $h = 0$. For $r > 0$, instead, one finds an interval $-h_0 \leq h \leq h_0$ in which the parameter difference becomes negative with minimum value $(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4) = -4C^2/9$ at $h = 0$. The size of this interval depends on the value of r and shrinks to zero, $h_0 \rightarrow 0$, as $r \rightarrow 0^+$.

3. Parameter behavior in the QCD phase diagram

The parameters $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ in Eqs. (9) and (10) are universal functions of the spin model variables in the scaling region. However, the mapping from r and h to μ_B and T in QCD thermodynamics is not universal but strongly model-dependent. By relating the density difference from the critical density in QCD to the magnetization M , this mapping has to satisfy, nonetheless, certain constraints near the critical point based on universality class arguments. The QCD critical point with baryon chemical potential μ_B^{cp} and temperature T^{cp} must be located at $r = h = 0$, positive r -values must correspond to the QCD crossover regime and positive h -values have to be realized in the denser phase.

In practice, these conditions can be assured by a simple linear mapping employing as auxiliary variables

$$\tilde{r} = \frac{(\mu_B - \mu_B^{\text{cp}})}{\Delta\mu_B^{\text{cp}}}, \quad \tilde{h} = \frac{(T - T^{\text{cp}})}{\Delta T^{\text{cp}}}. \quad (11)$$

The parameters ΔT^{cp} and $\Delta\mu_B^{\text{cp}}$ relate scales in the spin model coordinate system with the unknown size of the critical region in QCD. As by definition $\tilde{r} > 0$ in the QCD first-order phase transition regime, one has to rotate \tilde{r} to obtain r satisfying the above condition. Since the first-order phase transition line is expected to be bent, it is intuitive to perform this rotation such that the r -axis lies tangentially to the transition line in the QCD critical point. The exact orientation of the h -axis is, in contrast, less constrained. In line with [17], we opt for defining $h = \tilde{h}$ parallel to the T -axis in QCD.

The corresponding behavior of the scaled parameter $\tilde{\lambda}_3/C$ and the scaled parameter difference $(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)/C^2$ in the QCD phase diagram is shown in Figures 1 and 2, respectively. Qualitatively, the observed patterns follow the behavior with r and h described in Section 2, but appear tilted to some extent given the particular mapping to QCD thermodynamics employed in [17].

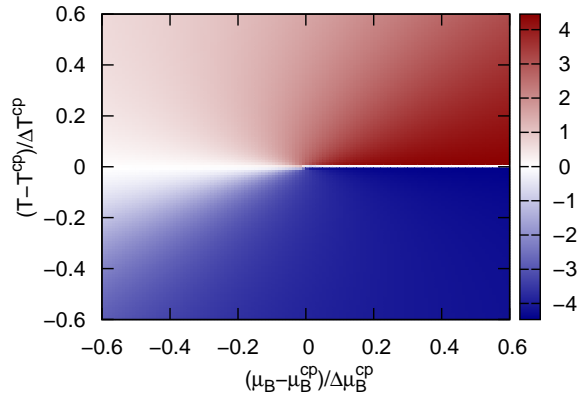


Figure 1. Scaled dimensionless parameter $\tilde{\lambda}_3/C$ from Eq. (9) as a function of \tilde{r} and \tilde{h} , cf. text for details.

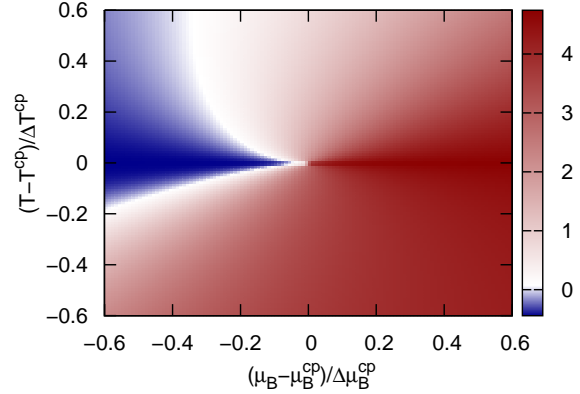


Figure 2. As in Figure 1 but for the scaled dimensionless parameter difference in Eq. (10), $(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)/C^2$.

4. Conclusions

We discussed the behavior of important parameters of the effective action near the QCD critical point in line with the three-dimensional Ising model universality class. The shown results are based on the mapping used in [17]. In future work, these parameters will serve as input for dynamical studies of critical fluctuations in heavy-ion collisions, similar to [18, 19].

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